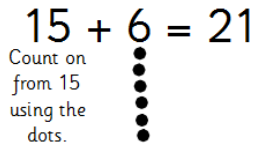
Addition

Children need to understand the concept of addition as combining two or more groups to give a total, and that the amount increases. They also need to understand and work with the principles that addition is the **inverse of subtraction**, that it is **commutative i.e. 5 + 3 = 3 + 5** and that it is **associative i.e. (5 + 3) + 7 = 5 + (3 + 7)**. The fact that it is commutative and associative means that calculations can be rearranged.

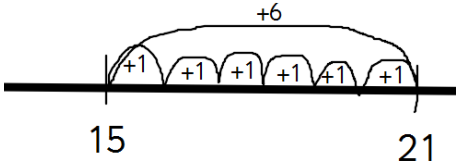
Year 1

Add one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

Children develop their ability to add by using practical equipment, combining groups of objects to find the total by counting all or counting on.



Children learn to draw their own number lines to find answers.



15 + 6 = 21

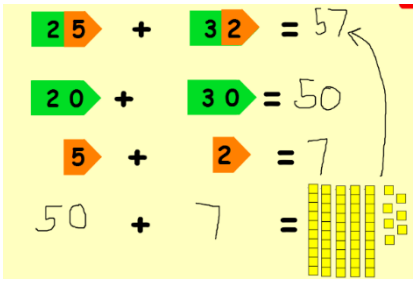
Children record their calculations as linear 'number sentences' e.g. 15 + 6 = 21.

Using their developing understanding of place value, they move on to use Base 10 equipment. They record their own calculations.

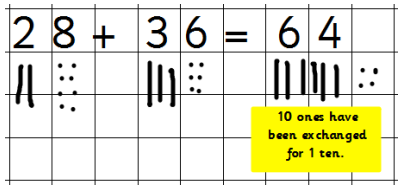
Year 2

Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one-digit numbers.

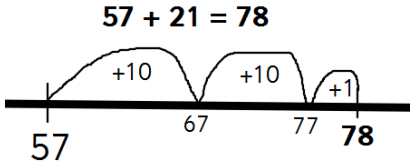
Children continue to use place value equipment to support their calculations. They learn to partition to add the tens first and then count on the ones.



Children record calculations using their own drawings of Base 10 (as lines for the 10 rods and dots for the unit blocks). When the units/ones total more than 10, children are encouraged to exchange 10 ones for 1 rod of 10.



Children draw their own number lines to add two-digit numbers.

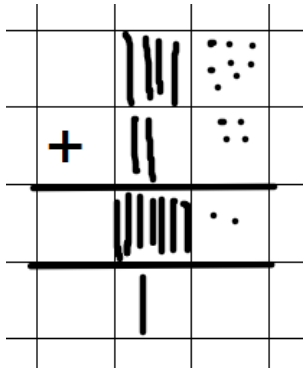


Column methods may be introduced when children are ready – the Y2 curriculum states, 'Recording addition in columns supports place value and prepares for formal written methods with larger numbers'.

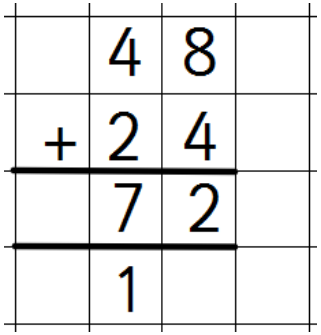
Year 3

Add numbers with up to three digits, using formal written method of columnar addition.

Base 10 can be drawn to support the children's understanding of place value.

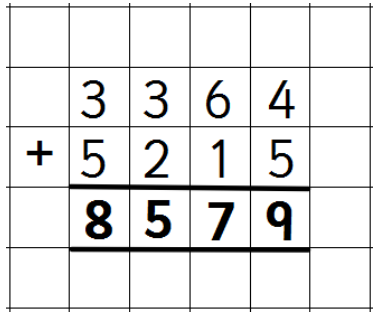
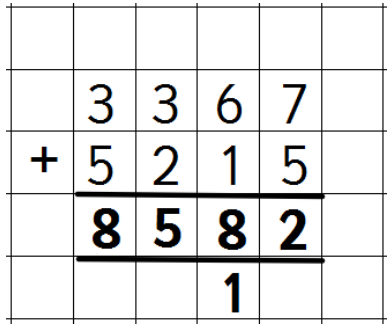
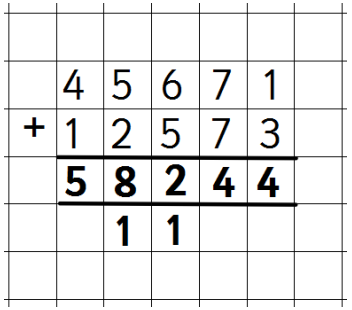
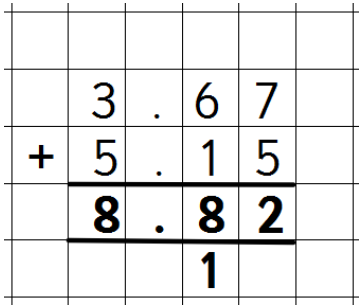
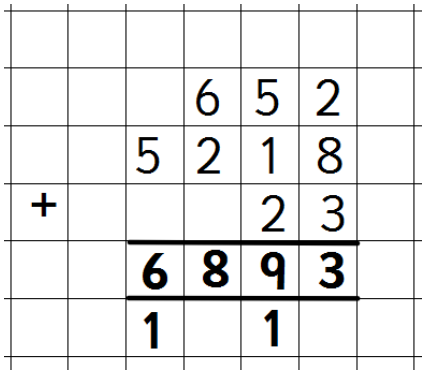
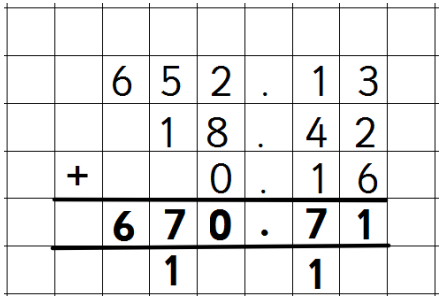


Practical methods are used alongside formal written methods where needed.



Decimals are only used in the practical context of money.

Addition

Year 4	Year 5	Year 6
Add numbers with up to four digits using the formal written method of columnar addition where appropriate.	Add whole numbers with more than four digits including formal written methods (columnar addition). Solve addition problems using decimal notation.	Solve addition multi-step problems in contexts.
Children continue to consolidate the formal written method for addition for numbers up to four digits.  Including carrying:  Decimals are only used in the context of money.	Children continue to use the carrying methods to solve calculations such as:  They also add decimals with up to two decimal places, knowing that the decimal points line up under one another. They are expected to add amounts of money and measures in different contexts. 	Children extend the carrying method and use it to add whole numbers and decimals with any number of digits, understanding the place value.  

Children are encouraged to solve real life problems including those with money and measures, and make decisions about which method is most appropriate.

Subtraction

Children need to understand the concept of subtraction as removal of an amount from a larger group (take-away) and comparison of two amounts (difference).They also need to understand and work with the principles that subtraction is the **inverse of addition**, that it is **not commutative i.e. 5 - 3 is not the same as 3 – 5** and that it is **not associative i.e. (10 – 3) – 2 is not the same as 10 - (3 - 2)**.

Year 1

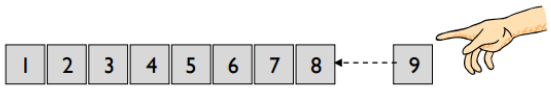
Subtract one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

Children use practical equipment such as counters and cubes from the Base 10 equipment.

13 – 4 =

Children touch count and remove the number to be taken away, in this case 4.

Touch count to find the number than remains.



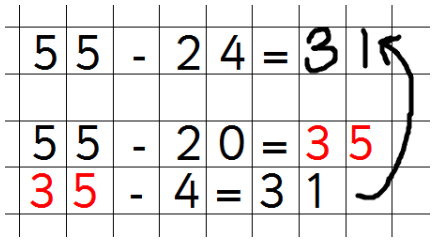
Children record their calculations as linear 'number sentences' e.g. 13 – 4 = 9.

Children use a number line and 100 square to support their understanding of subtraction.

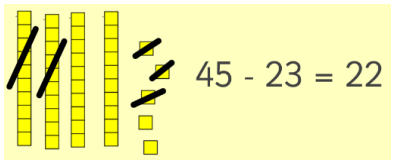
Year 2

Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one-digit numbers.

Children use partitioning to subtract the tens and then the units/ones.

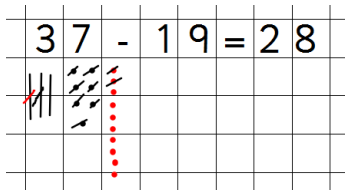


Alongside this, children use the Base 10 equipment to support their calculations. <u>They need to remove the units/ones and then the tens.</u>

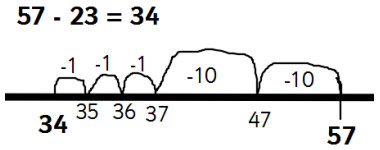


Children also record calculations using their own drawings of Base 10 (lines for the 10 rods and dots for the unit blocks).

In this case there are not enough units/ones to remove the 9 so they need to **exchange** a ten into ten ones. Y2 call this swap shop! The mathematical term for this is **decomposition**.



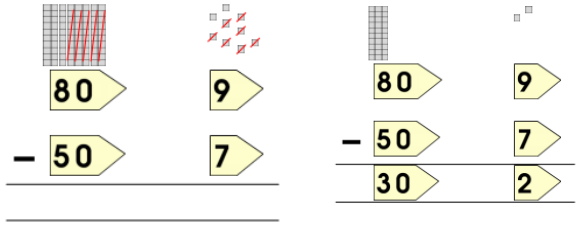
Children draw their own number lines to find the answer.

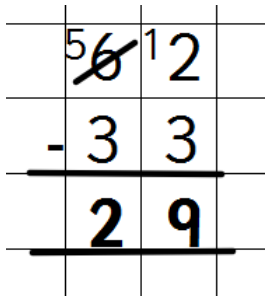


Year 3

Subtract numbers with up to three digits, using formal written method of columnar subtraction.

Practical methods should be used alongside formal written methods where needed.





62 = 60 + 2

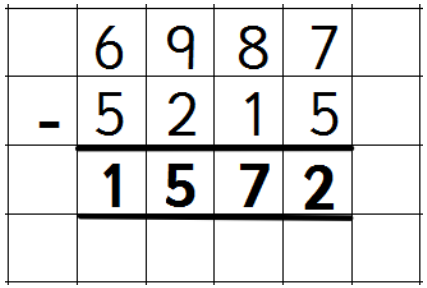
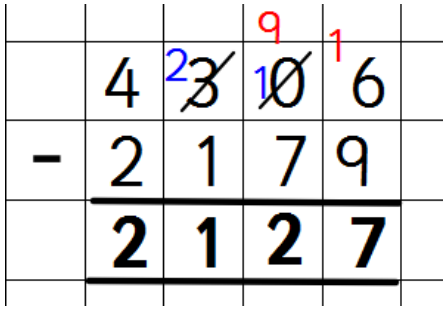
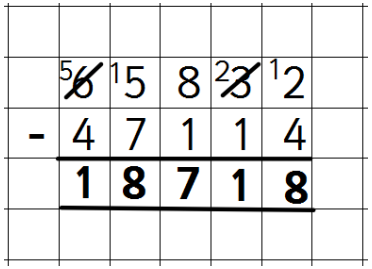
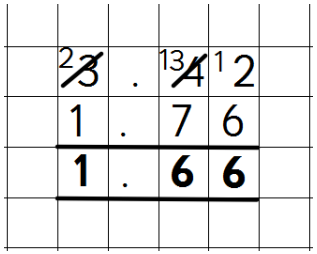
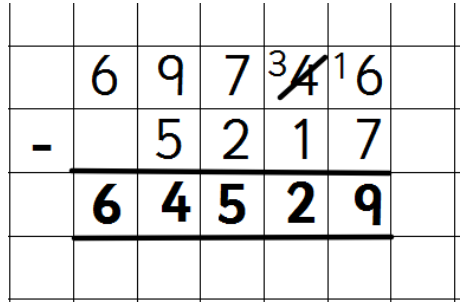
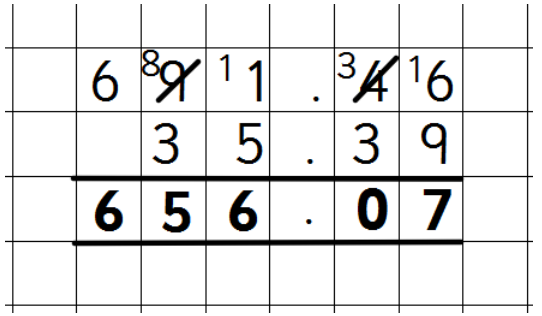
= 50 + 12

It is important for children to recognise the value of each digit when exchanging. In this example, 62 has become 50 + 12 so the calculations required are 12 – 3 and 50 – 30 (not 5 -3). Children start with the unit/ones column first.

By the end of Y3, children extend this method to three digit numbers.

Children draw their own number lines to find the answer. They can also use this method to find the difference.

Subtraction

Year 4	Year 5	Year 6
Subtract numbers with up to four digits using the formal written method of columnar subtraction where appropriate.	Subtract whole numbers with more than four digits including formal written methods (columnar subtraction). Solve subtraction problems using decimal notation.	Solve subtraction multi-step problems in contexts.
Children continue to consolidate the formal written method for subtraction for numbers up to four digits.  Including decomposition:  Children may draw a number line to find the difference.	Children continue to use the decomposition methods to solve calculations such as:  Children also subtract decimals with up to two decimal places, knowing that the decimal points line up under one another. They use this method to subtract amounts of money and measures.  Children may draw a number line to find the difference.	Children extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits, understanding the place value.   Children may draw a number line to find the difference.

Children are encouraged to solve real life problems including those with money and measures, and make decisions about which method is most appropriate.

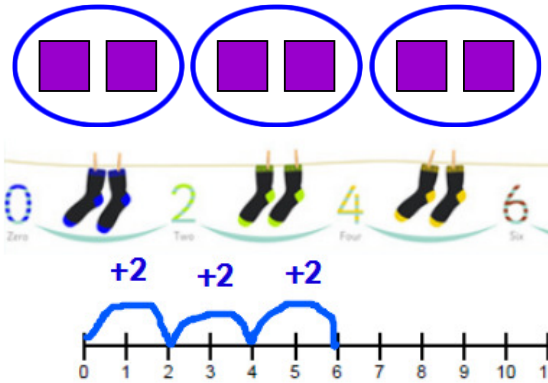
Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication as repeated addition. They should also be familiar with the fact that it can be represented as an array. They also need to understand and work with certain principles, i.e. that it is **the inverse of division**, and **commutative** i.e. 5 x 3 is the same as 3 x 5, and **associative** i.e. 2 x 3 x 5 is the same as 2 x (3 x 5).

Year 1

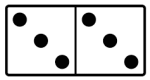
Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Children count on and back in 2s, 5s and 10s and look for patterns.



Children continue to solve multiplication problems using practical equipment and jottings, using the equipment to make groups of objects.

Children see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc. and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?



Dominoes and dice are used to reinforce doubles.

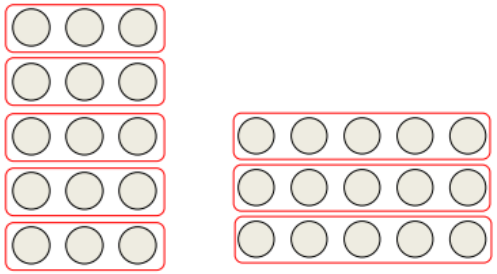
Year 2

Calculate mathematical statements for multiplication (using repeated addition) and write them using the multiplication (x) and equals (=) signs.

Children recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.

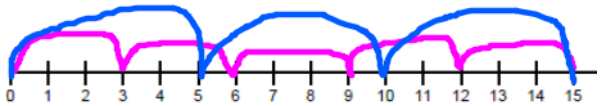
5 x 3 can be shown as 5 groups of 3.

5 x 3 can be represented as an array in two forms (as it has commutativity):



Children record their calculations as linear 'number sentences' e.g. **5 x 3 = 15** and **3 x 5 = 15**

This can also be shown on a number line.



Children link doubling to multiplying by 2.

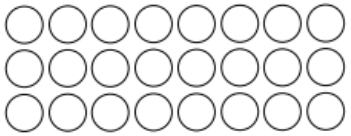
Year 3

Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, progressing to formal written methods.

Children recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.

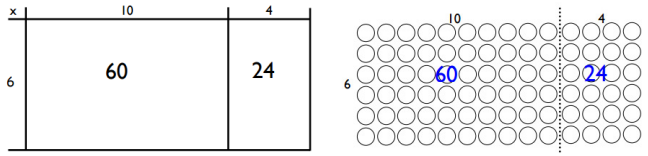
Children continue to use arrays where appropriate, linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10) e.g. 3 x 8.

They use practical equipment to solve problems.



3 x 8 = 8 + 8 + 8 = 24

As they progress to multiplying a two-digit number by a single digit number, children use their knowledge of partitioning two digit numbers into tens and units/ones to help them. For example, when calculating 14 x 6, children can partition the 14 into 10 and 4.



Partitioning in this way allows children to identify that the first array shows 10 x 6 and the second array shows 4 x 6. These can then be added to calculate the answer:
(6 x 10) + (6 x 4)
= 60 + 24
= 84

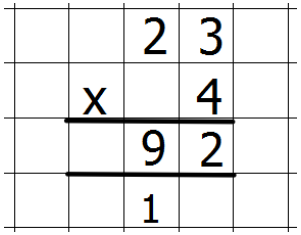
Multiplication

Year 4

Multiply two-digit and three-digit numbers by a one-digit number using the formal written method of short multiplication.

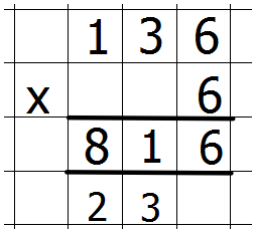
Children recall multiplication and division facts for multiplication tables up to 12 × 12.

Children are taught to carry **below** the answer.



4 x 3 = 12
The 2 ones/units are recorded in the ones/unit column. The 10 is recorded below the answer bar to show 1 ten is being carried into the tens column.
4 x 20 = 80
8 tens and 1 ten make 9 tens. So 9 is recorded in the tens column.

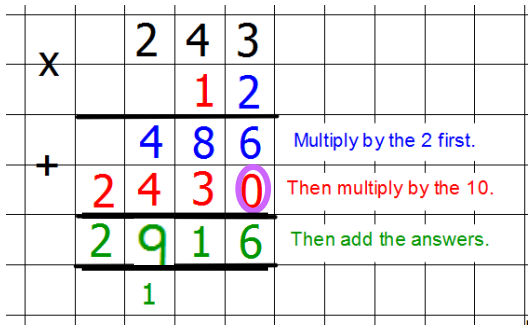
Here is an example of a three-digit number multiplied by a one- digit number.



Year 5

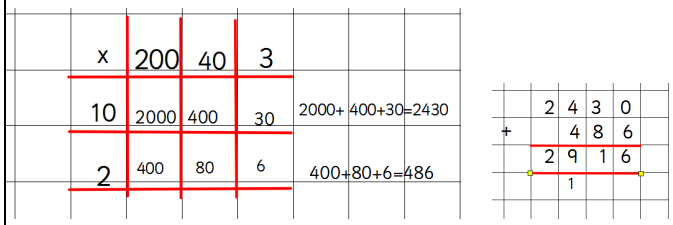
Multiply numbers up to four digits by a one- or two-digit number using a **formal written method, including long multiplication for two-digit.**

Once children are secure with short multiplication from Year 4, they move on to multiplying up to four digits by two digits using long multiplication.



Children are taught to multiply by the ones first, then multiply by the tens number and finish by adding together the totals using column addition.
When multiplying by the tens they are taught to put the place value holder in first (circled above).

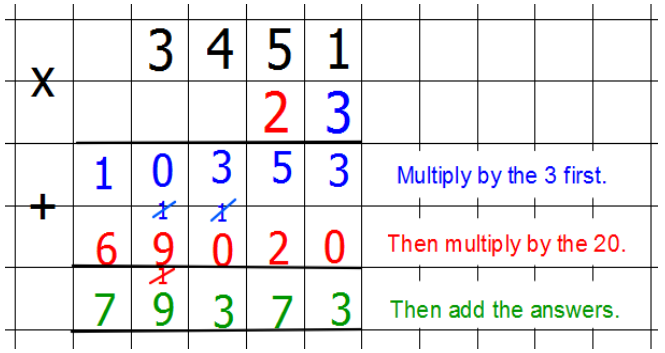
Although not a method stated in the NC Mathematics Appendix 1, if needed, the grid method below can be used for extra clarification.



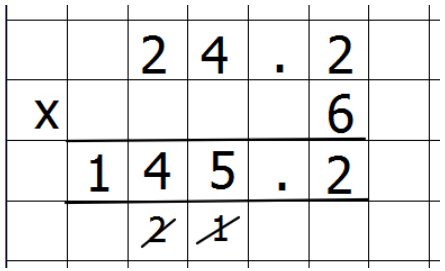
Year 6

Multiply multi-digit numbers up to four digits by a two-digit whole number using the **formal written method of long multiplication**.
Multiply numbers with up to two decimal places by whole numbers.

Children move onto multiplying four digits by two digits continuing to use the long multiplication method.



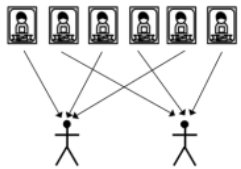
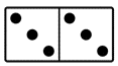
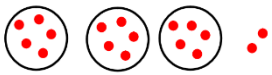
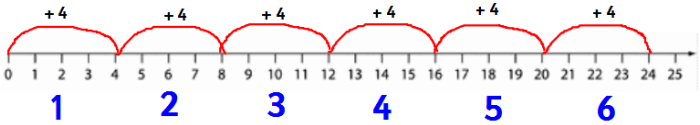
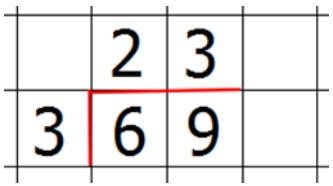
Children use this method to multiply by decimal numbers with up to two decimal places.



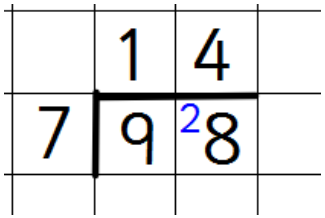
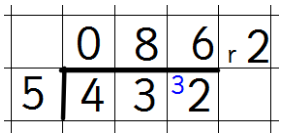
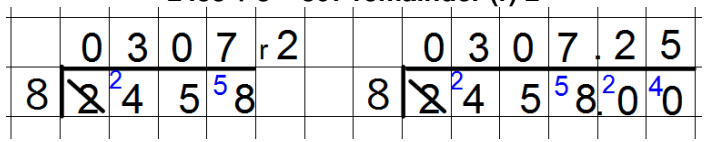
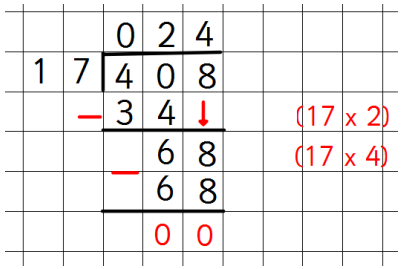
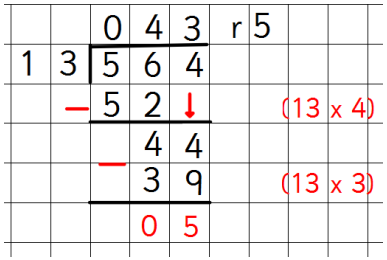
Children are encouraged to solve real life problems including those with money and measures, and make decisions about which method is most appropriate.

Division

In developing a written method for division, it is important that children understand the concept of division as repeated subtraction. They also need to understand and work with certain principles, i.e. that it is **the inverse of multiplication**, **not commutative** i.e. 15 ÷3 is not the same as 3 ÷ 15, and **not associative** i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷ 2.

Year 1 Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.	**Year 2** Calculate mathematical statements for division within the multiplication tables and write them using the division (÷) and equals (=) signs.	**Year 3** Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, progressing to formal written methods.
Children learn to count on and back in 2s, 5s and 10s and look for patterns. Children solve division problems using practical equipment and jottings. They use equipment to share objects and separate them into groups, answering questions such as 'If you **share** six apples into three **groups**, how many will you have in each group? How do you know?' or 'If six football stickers are **shared between** two people, how many do they get each?'  They may solve both of these types of questions by using a 'one for you, one for me' strategy until all of the objects have been given out. Children should be introduced to the concept of **simple remainders**. This can be done when children identify that groups are not equal and the remainder is referred to as the part that is **left over**. Dominoes and dice are used to reinforce doubles and halves. 	Children recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers. The relationship between multiplication and division is continually referred to. Children use pictorial representations to share objects into groups, using their own jottings to support them. This shows 15 counters organised into **groups of** 5. 15 ÷ 5 = 3  Children also continue to develop their knowledge of division with remainders. This shows 17 counters organised into groups of 5 with two counters left over. This is recorded as 17÷ 5 = 3 r2  Children need to be able to make decisions about what to do with remainders after division, and round up or down accordingly. In the calculation 17 ÷ 5, the answer is 3 remainder 2, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below: • I have £17. Books are £5 each. How many can I buy? Answer: 3 The remaining £2 is not enough to buy another book. • Apples are packed into boxes of 5. There are 17 apples. How many boxes are needed? Answer: 4 The remaining 2 apples still need to be placed into a box.	Children recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables e.g. 5 x 8 = 40 so 40 ÷ 8 = 5 Children become fluent at using multiplication facts to solve related division problems. To solve 24 ÷ 4, children are encouraged to use their knowledge of the 4 times table. They may record jottings/number lines to show counting in 4s: 4, 8, 12, 16, 20, 24, – this shows that the answer is 6 because 6 lots of 4 are needed to get up to 24.  Children begin to use the short division method for calculations with exact answers. This method is sometimes referred to as the "bus stop" method. **69 ÷ 3 = 23** Step 1 60 ÷ 3 = 20 Step 2 9 ÷ 3 = 3 Final answer = 23  Children solve real life problems including those with money and measures and continue to make decisions about what to do with remainders after division (see Year 2).

Division

Year 4	Year 5	Year 6
Become fluent in the formal written method of short multiplication and **short division** with exact answers	Divide numbers up to four digits by a one-digit number using the **formal written method of short division** and interpret remainders appropriately for the context.	Divide numbers up to four digits by a two-digit number using the **formal written method of short division where appropriate**, interpreting remainders according to the context. Divide numbers up to four digits by a two-digit whole number using the **formal written method of long division**, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
Children recall multiplication and division facts for multiplication tables up to 12 × 12. Children practise to become fluent in the formal written method of short division for calculations with exact answers. **98 ÷ 7 = 14**  In this short division take the first digit (9) of the number and divide it by 7. 9 ÷ 7 = 1 remainder 2. Carry the remainder into the next column. This makes a new two digit number: 28. 28 ÷ 7 = 4. The answer is 14.	Children should be encouraged to use higher multiples of the divisor. Children use the formal written method of short division for calculations that include remainders. **432 ÷ 5 = 86 remainder (r) 2**  In this short division, take the first two digits (43) of the number and divide them by five. 43 ÷ 5 = 8 remainder 3. Carry the remainder into the next column. This makes a new two digit number:32 32÷ 5 = 6 remainder (r) 2 Children are taught to record the remainder as a fraction or a decimal. **2458 ÷ 8 = 307 remainder (r) 2**  The remainder can then be recorded as a decimal 2458 ÷ 8 = 307.25 or a fraction 2458 ÷ 8 = 307 and ¼ (2/8 written in its simplest form). When children are secure in short division then the formal method of long division can be introduced.	Children use the formal written methods of short and long division. Below is an example of long division. 408 ÷ 17 = 24  For children to receive marks in the arithmetic paper they **must** include the zero and complete the calculation fully. The jottings at the side are not required but children are encouraged to write jottings if they need to.  Children may be required to show the remainder as a fraction in its lowest form or a decimal. e.g. As a fraction ⟶ 564÷ 13 = 43 and 5/13 As a decimal (rounded to the nearest dp) 43. 4

Children are encouraged to solve real life problems including those with money and measures, and make decisions about which method is most appropriate.